Dispersive e. m. Corrections to πN Scattering at Threshold

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Abstract. The e. m. dispersive corrections to the πN scattering lengths are derived using minimal e. m. coupling in PCAC for the nucleon and Δ pole terms in the heavy baryon limit. Form factors and masses are assumed to have their empirical values, with no free parameter. This approach gives a large correction to the elastic charged-pion isoscalar scattering length. The result is compared to that of chiral effective field theory (EFT) and applied to the $1S$ energy shift of the $\pi^- p$ atom.

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For purely strong interactions the πN scattering amplitudes at threshold are fundamental quantities which enter into the discussion of various problems. They provide a basic test of the Tomozawa-Weinberg chiral relation for the isovector and isoscalar scattering lengths a^- and a^+, respectively, in the limit $\omega \to 0$ with $a^- = \omega/(8\pi F_\pi^2)$ and $\simeq 0.089\ m_\pi^{-1}$; $a^+ = 0$, where $F_\pi \simeq 93\ MeV$ is the pion decay constant and $\omega = m_\pi$ at threshold. Second, the empirical isovector scattering length is the main ingredient and uncertainty in the GMO dispersion relation by which the πNN coupling constant 14.11 ± 0.05 is determined[1] etc. The major precision source for these quantities are the remarkable measurements of the 1s level shifts in pionic hydrogen and deuterium as well as the corresponding widths. For pionic hydrogen the precision is 0.2% for the shift and 4% for the width [2, 3]. Since the (complex) energy shift relates exactly to order α^2 to the so-called Coulomb scattering length by $\varepsilon_{1s} \propto \phi^2_{Bohr}(0)a_C$, these quantities convert to similar precision for the hadronic scattering length a_h but for e. m. corrections.

Such corrections to the amplitude can be evaluated either using chiral effective field theory (EFT) [4, 5, 6] in terms of a systematic momentum expansion in orders $\mathscr{O}(p^n)$ and α or, alternatively, in terms of a complementary, less ambitious but more physical, wave function approach, which we are presently developing [7, 8]. The latter assumes that the hadronic interaction is of short range and that the e. m. form factor determining the charge distribution is empirically known. In the heavy baryon limit, gauge invariance and the known low energy expansion of the πN amplitude give then the correction provided the hadronic interaction is treated as an effective Lagrangian one. This gives 3 natural e. m. correction terms for the $\pi^- p$ case in terms the hadronic scattering length a_h [7]:

1. The correct wave function at $r = 0$ for the extended charge distribution: $(\delta a_h/a_h)_{wf} = -2\alpha m_\pi \langle r \rangle_{em} \simeq (-0.9\%)$.
2. The correct final state wave function of the extended charge distribution (cusp): $(\delta a_h/a_h^2)_{cusp} = -8\pi\alpha\ [2 - \gamma + \log 2\alpha - \langle \log m_\pi r \rangle_{em}] \simeq (+0.7\%)$.

3. The correct interaction energy at $r = 0$ in the Coulomb field of the extended charge distribution by gauge invariance; the parameter b_h describes the energy dependence: $(\delta a_h)_{gauge} = (-\alpha\langle\frac{1}{r}\rangle_{em})b_h \simeq (-1.0\%)a_h$

These corrections are exact to order $\mathscr{O}(\alpha^2)$ in the limit of a short ranged strong interaction and no inelastic intermediate states. Here we generalize to the problem by including inelastic intermediate states from processes $\pi N \to \gamma X$. A guide to the importance of such terms is the dominant contribution of the Kroll-Ruderman radiative capture process $\pi^- p \to \gamma n$ at threshold [9], which leads to an observed $1s$ width which is 8% of the observed $1s$ interaction shift, a huge number.

The matrix element for this radiative capture can, for example, be derived from the Partially Conserved Axial Current (PCAC) relation using minimal e. m. coupling

$$\partial_\mu A_\mu = -m_\pi^2 F_\pi^2 \phi_\pi(x);\ \partial_\mu \to \partial_\mu \pm ie\mathscr{A}_\mu, \tag{1}$$

where $\mathscr{A}_\mu$ is the e. m. 4-vector potential. This corresponds to electric dipole (E1) radiation due to the discontinuity in the current. In the heavy baryon limit and at threshold the radiation comes only from the vertex itself .

The characteristic features of the contribution from $\pi^\pm N \to \gamma N' \to \pi^\pm N$ are:

-the transition is an axial one with a a well defined strength.

-the axial form factor $F_A(\vec{q}^{\,2})$ is well approximated by a dipole shape.

$F_A(\vec{q}^{\,2}) = (1+\vec{q}^{\,2}/M_A^2)^{-2}$ with $M_A = (960 \pm 30)\,\mathrm{MeV}$.

- typical energy denominators $(p \pm m_\pi)^{-1}$ appear from intermediate states with the sign switch due to crossing. For the $\pi^\pm p$ case

$\left(1+\frac{m_\pi}{M_N}\right)\delta a_{1s}^{(N'\gamma)} = \frac{3\alpha}{8\pi^2}\frac{g_A^2}{F_\pi^2}\mathscr{P}\int_0^\infty \frac{dp\,p\,F_A^2(p^2)}{p \pm m_\pi - i0}.$

In the soft limit $m_\pi = 0$ this results in a large nucleon isoscalar term of 3%. However, the Δ isobar and the nucleon are basically identical, but for the $N\Delta$ mass splitting and weight factors. It is unnatural to include the nucleon only. In the limit of no $N\Delta$ mass splitting, the additional Δ intermediate states lead to a multiplicative factor $25/9$ as compared to the result with the intermediate nucleon only. This increases the previous 3% to 9%, an enormous correction! Why is this so large? The reason is that the scale parameter is the axial mass M_A and not the pion one m_π which gives a factor 7 enhancement. When the $N\Delta$ mass splitting of $\approx 2m_\pi$ is brought in, it cuts the $\gamma\Delta$ term by 50% to a total isoscalar correction of about $+6\%$. This is still very large, but we expect relativistic kinematic factors to cut it additionally to about 4.8%.

If $m_\pi \neq 0$, new characteristic terms appear of the type $c\,m_\pi \ln(m_\pi/M_A) + d\,m_\pi$, These are generated both from the nucleon and the Δ intermediate states and they depend only weakly on the exact value of M_A. In the particular case of the nucleon intermediate state, the term proportional to $m_\pi \ln m_\pi$ has the identical coefficient to the one found to third order in chiral EFT by Gasser et al. [6]. If we expand the amplitude $\delta a_{1s}^{(n\gamma)}$ for the $\pi^- p$ case in in terms of the small parameter $x = x_\pi = m_\pi/M_A$, we have:

$\left(1+\frac{m_\pi}{M_N}\right)\delta a_{1s}^{(n\gamma)} = \frac{3\alpha}{8\pi^2}\frac{g_A^2}{F_\pi^2}\left[\frac{5\pi}{32}M_A - m_\pi\left(\ln\frac{m_\pi}{M_A} + \frac{11}{12} + \mathscr{O}(\frac{m_\pi}{M_A})\right)\right].$

When the Δ isobar is degenerate with the nucleon, the nucleon pion-mass term is strongly canceled, such that the dependence on the pion mass becomes negligible. However, with the empirical $N\Delta$ mass splitting, the Δ pion-mass contributions are quenched such that the contribution from the nucleon result is partly restored.

One can now compare these results to those obtained using an effective chiral Lagrangian [4, 6] to leading order. We only sketch a comparison of some specific points. It is important to realize that certain of the predictions of such an effective field theory are specific to EFT and outside our present approach, while, on the contrary, our approach automatically generates some terms which require higher order in the EFT expansion. In the heavy baryon limit, the e. m. isospin breaking in the πN threshold amplitudes are related to the the e. m. mass of the nucleon and the np e. m. mass difference in the EFT beyond the purely kinematic effects [5]. To next-to-leading order these relations can be expressed in terms of 3 unknown constants $f_{1,2,3}$; we obtain model predictions for f_1 and f_2. Using $\pi^{\pm}p$ elastic scattering for illustration gives the following relations

$$M_n^{em} = -e^2F_\pi^2\Big[f_1+f_3\Big]\ ;\ M_p^{em} = -e^2F_\pi^2\Big[f_1+f_2+f_3\Big]\ ;\quad a_{\pi^\pm p}^{em} = -2\pi\alpha[f_1 \pm \tfrac{1}{4}f_2].$$

The dispersive contribution from intermediate $\gamma N(\Delta)$ states to the scattering length is isoscalar in the charged pion sector in the limit of a vanishing pion mass and has the symmetry property of the contribution in EFT by the next-to-leading order constant f_1. This constant appears as well a part of the e. m. neutron mass M_n^{em}, but then it always comes in the combination f_1+f_3. Such terms cannot be physically separated [5]. Our value with the physical Δ isobar included gives $F_\pi^2 f_1 = -28(1)\ MeV$ Dimensional estimates inside of EFT give intermediate estimates $F_\pi^2|f_1| = 6\ MeV$ and $12\ MeV$ [5, 6]. There appears therefore to be little relation of this parameter with the neutron e. m. mass, which suggests a massive cancellation between the EFT constants f_1 and f_3.

Isospin violation in πN elastic scattering has been shown here to have well determined contributions originating in the Coulomb field of the extended charge with little model dependence. These corrections are general and involve terms beyond present EFT approaches. To next-to-leading order in the chiral expansion our terms have counterparts in EFT. The dispersive term with $\gamma N(\Delta)$ intermediate states gives an important isoscalar isospin breaking term which can be viewed at as the major model contribution to the EFT constant f_1. For a non-vanishing pion mass, the same mechanism generates a small and model-insensitive isospin breaking in the isovector interaction consistent with the finding of Meissner et al. [5]. A detailed conference presentation of the present material can be found in hep-ph/0504258.

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